Mapping children's mathematical development in the early years

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Abstract

The tasks described here, and those still under development,are in a highly structured format but the look and feel of the tasks, from the children's perspective, is that of games. These games cover a range of aspects of mathematics, including number, chance, measurement, and mathematical structure. All games use simple equipment. An over-arching feature that these games have is that they should begin play with concrete materials, and then move on to playing mentally.
To date several children have been interviewed and issues with the games revealed. Two examples of games are detailed, and interested parties are invited to comment or participate in further development.

Introduction

Several reasons have been put forward for the importance of a good understanding of children's mathematical development in the years prior-to-school. These include the increasing number of children who are participating in early childhood programmes (Doig, McCrae, & Rowe, 2003) and the recognition of the importance of mathematics (Doig et al., 2003). Further, there is concern about the difference between the development of children from different socio-economic standards and ethnic groups (Thomson, Rowe, Underwood, & Peck, 2005). These, inter alia, have raised the need for better assessment of children's mathematical development prior-to-school.

Being researchers in the fields of both mathematics and early childhood development, we are currently engaged in designing an assessment instrument to elicit young children's developing mathematical understandings. The target group of children are those in the three years prior-to-school: that is,they have had no formal education in mathematics. In Australia this is ages 2 to 4 in some jurisdictions (Tasmania, New South Wales, Victoria, South Australia and the Australian Capital Territory) and ages 3 to 5 in the remaining states and territories.

The goal of this endeavour is to provide researchers and early childhood practitioners with a tool that provides a mapping of typical development, provides normative interpretations of a child's responses, and, most importantly, makes possible the description of a child's responses in plain language for communication between researchers and early childhood practitioners, between early childhood practitioners, and between early childhood practitioners and parents.

In short, our intention is to construct mathematical tasks that engage children and provide evidence of mathematical development, together with a described continuum of this development that enables early childhood educators to gauge and address children's mathematical needs, and supports communication for, and between, all stakeholders.

Existing assessment tools

Given the suggested importance of early development in mathematics (B. Doig et al., 2003)), the number of currently available assessment tools for mathematical development in the early years is remarkably small. Clements and his colleagues (2008) reviewed a range of existing instruments, including the well-known Woodcock–Johnson III (Woodcock, McGrew, & Mather, 2001) and the Bracken Basic

Concept Scale (Bracken, 1998), but advise caution when using either of these as the sole measure of a child's mathematical development, while the Test of Early Mathematics Ability (Ginsburg & Baroody, 2003), is reviewed more positively. An interesting point to note is that the available instruments tend to have an emphasis on number, despite the fact that most educators in the early years field maintain that children of this age-group are observed, daily, 'doing' a wide range of mathematical activity (Hunting et al., 2008).

Assessment instruments developed by one of the present authors were also reviewed. *Who am I?* (de Lemos & Doig, 1999) is composed of a set of tasks to which the child responds by writing or drawing. These responses are categorized by comparison with a sample set of responses that have been Rasch analyzed. Results are presented in a graphical form of report. The assessment tasks include:

- My name is … where the child is asked to write their name;
- I can draw a circle … where the child is asked to copy a provided circle;
- I can write numbers … where the child is asked to write some numbers;
- This is a picture of me … where the child is asked to draw themselves.

Other mathematical tasks asked for a copy of a square, triangle, cross, and a diamond shape. Other, non-mathematical, tasks included writng letters, words, and a sentence.

This instrument was designed to assess what is popularly known as 'school readiness' but has been used also in a large-scale study of Australian children's mathematical development in the year before they entered school and in their first year of school, *Project Good Start* (Doig et al., 2003). Children were assessed in their first year at school using *I can do maths* (Doig & de Lemos, 2000) which focusses on those aspects of the mathematics curriculum relevant to the first years of school. The scales from these two instruments were later combined by Rowe into the *Pre-school Numeracy Scale* (Thomson et al., 2005) for use in *Project Good Start*.

In summary, it appears that the development of an assessment instrument that gives due emphasis to the full range of young children's mathematics, and maps development of this range, is long overdue.

Background

De Lange suggested that before entering formal education young children have a 'sparkle', a curiosity, about scientific phenomena that, for many, appears to dissipate as they progress in formal education (De Lange, 2008). The Freudenthal Institute's project *Talentenkracht* (Curious Minds) seeks to explore this sparkle, using engaging tasks based on commonly available materials. The success of this approach suggests a possible path for developing tasks for young children to explore aspects of mathematics.

The range of mathematical topics explored were, to some extent, also in response to comments made by practictioners in the Hunting, Bobis, Doig, et al, study (2008). The topics are:

- Chance: randomness
- Number: counting, doubling
- Space: symmetry, shape, location
- Pattern: visual, aural
- Measurement: length, density
- Structure: classifying, logical thinking.

A further impetus was the work of Aubrey (1993) who suggested that the major facets of young children's informal mathematics that appear to offer productive pathways in future learning included measurement, geometry, mathematical thinking, pattern, representation, number, and mental imaging.

Task criteria

There are five criteria that were applied to tasks that were considered for selection, or were designed for inclusion, in the trial phase of development of the assessment instrument. These, too, are supported by Aubrey's (1993) suggestions, which include that tasks not be necessarily linked to school curriculum, use observations with structured tasks, and importantly, be scaled and reported against a descriptive scale, all of which we have set out to achieve, even if belatedly. Our criteria are set out below.

Criterion 1: Elicit a wide range of responses

The experience gained in developing *Who am I?* (de Lemos & Doig, 1999) suggested that tasks for assessment need to allow for a range of responses, which are not completely correct necessarily, but are partially correct. The categorization of these responses and the use of Master's Partial Credit Model (PCM) (1982) allows the construction of a continuum, a map, of development rather than simply mastery, and also to report results on a described continuum of development. An example of this approach to asssessment is to be found in *Tapping Student Science Beliefs* (Adams, Doig, & Rosier, 1991; Doig & Adams, 1993) where partially correct responses to science questions were categorized and subjected to a PCM analysis.

Criterion 2: Engage children

Young children in prior-to-school settings are used to choosing their play activities and how long that they attend to these activities. Thus, in the case of administering an assessment instrument, engaging a child in the assesment tasks and encouraging a willingness to respond, is a critical criterion for selecting tasks. One strategy is to use assessment tasks that are formatted as games, as games are seen as fun and are interesting to most children.

Criterion 3: Are open-ended

A third criterion is that the tasks should allow for a wide range of mathematical understandings. Research shows that young children have a wide range of depth in their capabilities in mathematics (see, for example, Aubrey, Godfrey, Kavkler, Magajna, & Tancig, 2000; Munn, 1998). Suitable tasks should allow all children to answer initially, and then progressively explore the extent to which the child has understanding: this type of task is the so-called 'ramped' task.

Criterion 4: Explore different types of understanding

Children come to know mathematics in different ways and to different extents. It would seem sensible then, to offer children the opportunity to solve tasks in different ways (see Heirsdfield and Lamb (2006a, 2006b) for reports of a case sudy of this idea in the early years of school). In discussion with colleagues at the Freudenthal Institute[1], suggestions were made to the effect that Hans Freudenthal's perspective is that one should start with solving a mathematical task with a concrete method, and move towards being able to use a mental method. We abbreviate this criterion as 'moving from concrete to mental'.

Criterion 5: Ease of use

Ease of administration is also a requirement. Simple to administer and score, clear guidelines for making inferences, and reporting that clearly and reliably shows what

[1] We acknowledhe the contribution of Marja van den Heuvel-Panhuizen and Anne van Hoogmoed in offering this Freudenthal approach, and reviewing early versions of some tasks.

the child can do. Time involved is also a factor with young children, and simplicity of use aids in keeping time to a minimum.

Reporting

The reporting of results is, as Aubrey (1993) points out, is a critical factor in the usefulness of any assessment tool. In the case of the assessment tool under development, it is planned to employ a Masters' Partial Credit Model analysis to the children's response data, in manner similar to that used in Doig and Adams; *Tapping Students Science Beliefs* (1993), where a described scale for children's understanding in science was developed. This approach provides a described continuum of development rather than a set of stages or levels. This is an important characteristic of an Item Response theory (IRT) analysis, as it shows the non-monotonic development of children's learning of mathematics. This approach suggests that the development of learning is more like golf, erratic, but goal driven, than it is like archery, a simple trajectory. That is to say, that this analysis will assist those charged with developing a mathematics curriculum for young children, in that it will not suggest a straight, monotonic development, but rather suggest that there may be a number of learning paths to a desired goal.

Two sample games

The two games described below are typical of the format of the games pilotted for the suite of games being developed for the assesment of young children's mathematical ability. Although each has a different focus in terms of the mathematics required, they both fit the criteria set out above. In particular, the Gum-nut game allows the child to continue until it becomes too dificult for them. In addition, the game can easily be varied for more advanced children to have them use imagined gum-nuts, rather than real objects.

In the Street game, it is envisgaed that a larger grid, with more combinations, could be used to extend the game to cater for children with more advanced structural competence.

Note that at this stage, no effort has been made to enjoin children to represent their experience or thinking. This aspect will be the subject of further trials in a later phase of development.

The Gum-nut game

In this game number game we use simple, natural counters. In Australia, gum-nuts, the seed pods of the eucalyptus tree, are easy to find but any simple counter can be used. If the child is very young we could start with two gum-nuts, but experience suggests that three is an ideal starting point from two-year olds and upwards.
The game is played in four stages, and these are described below.

Part 1

Start by showing the gum-nuts to the child.

Can you tell me how many gum-nuts I have?

If the response is not three, they are asked to count. If the child cannot agree that there are three gum-nuts, then either two gum-nuts are used, or the game abandoned.

This first phase of the game establishes that the child has the basic understanding, of an amount of objects, which shows whether the game is appropriate or not for this child.

Part 2

I am going to hide the gum-nuts in my hands.

This is done behind one's back or by turning away from the child.

Now show your closed hands to the child.

Which hand do you want to see opened? When the child points or tells, open that hand.

How many gum-nuts are there?

Wait for the child to respond.

How many gum-nuts are there in my other hand?

If the child is correct repeat the process, changing the number of gum-nuts in each hand. If the child is incorrect open both hands and have them count the number of gum-nuts in each and as well as the total number.

Part 3

Say to the child: *Now it is your turn. Don't let me see you hide the gum-nuts.*

After your turn to say how many in each hand, ask:

Was I correct?

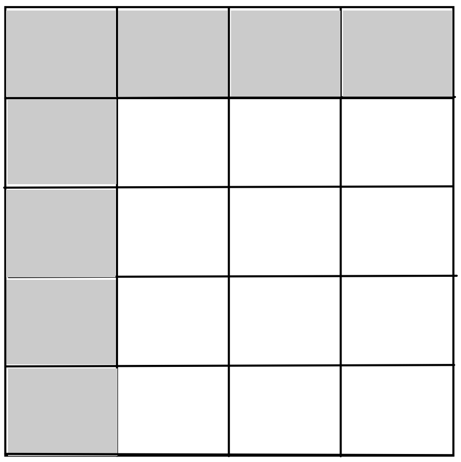
Ask the child: *How do you know?*

Part 4

If the child is fluent with a small number of gum-nuts, play again with 4 gum-nuts, then 5, until the task becomes too difficult or the child wishes to stop.

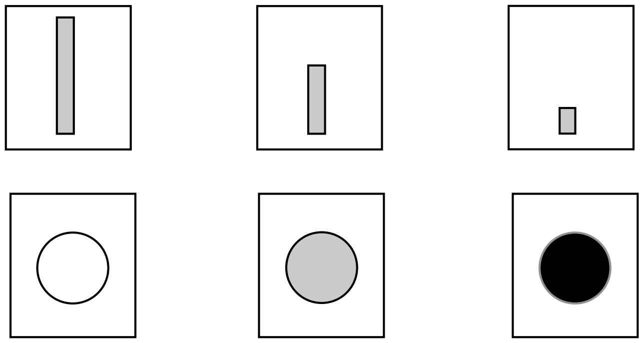
The Street Game

This game is played on a grid (4 by 4) with a border, as shown.
The rows and columns are the streets.
The shaded squares are for the street names: note that each street has a two-part names (eg., short-black street).

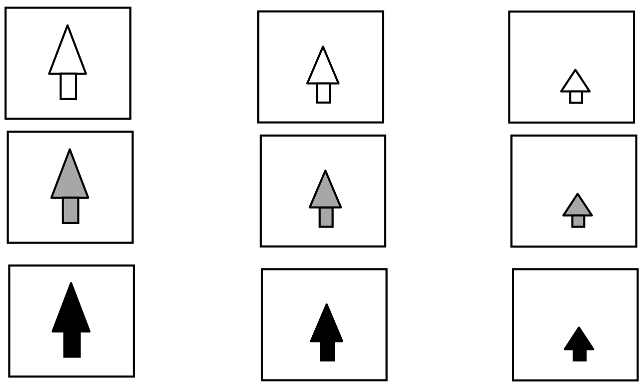


Playing pieces are as shown. The cards fit the grid squares.

The following cards are border cards (street names).



These tree cards are for placing in the streets.



Part 1

You should put the long, medium, and short cards across the top of the grid. Show the child what you are doing and ask them to tell you that one street is long street, one medium (middle) street, and the other short street. Now place the colour cards (white, grey, black) at the end of the rows. Explain that these colours also name the streets. Make sure that the child knows the street names.

Place one tree card into its correct square and explain why it is there. For example, the tall, grey, tree. Make sure that the child understands the street name idea, and then remove the tree card.
Ask the child: *Can you put one of these tree cards in its right street? Show me.*

If the child is correct and can name the streets and the tree (eg., the small black tree is in small street and black street) ask them continue with the remaining tree cards, otherwise repeat Step 1.
If the child does not understand after two tries at Step 1, stop.

Part 2

Put the long and short cards across the top of the grid in a different order to before. Place the colour cards in a different order at the end of the rows.

Ask the child: *Can you put one of these tree cards in their right street? Show me.*

If the child is correct, continue with the remainder of the tree cards, otherwise repeat the first part of Step 2. If the child does not understand after two tries, stop.

Part 3

Put two tree cards in the squares in two different streets.

Ask the child: *Can you put the other cards in their right places? Show me.*
Ask them to say the names of the streets (that is, which border card goes where).
If the child is incorrect on this part, repeat Step 3 from the beginning.
If the child does not understand after two tries, stop.

Part 4

If the child is correct, let them choose two tree cards to place, and you complete the game. Do not have the answer correct. Have them check your answer. Ask them to explain why you are incorrect.

Pilot Data

Pilot data have been collected for these tasks, and the results are encouraging. For example, in the Gum-nut game, a three-year-old said that she knew what was in the other hand because "because there were 2 in one hand and 1 more in the other hand and that is 3". Some responses, were, of course, less fullsome. These included "I can count" (2-year-old girl), and "Coz you got this one 3 (holds out a hand) and this one 2 (holds out the other hand) (3 year-old- boy). But all these children were correct in their responses.

Discussion

The scarcity of stimulating, reliable, and valid mathematics assesment for very young children has been described by researchers such as Clements (2008) despite the earlier outline of what good assesment shoud be like, and on what it could focus (Aubrey, 1993). The game environment used in the tasks described here would appear to be successful in engaging children's interest, a critical factor with the very young. Further, the approach used to develop and select games has provided a suite that covers a large extent of informal mathematics. Some games would appear to be providing little useful information (or there is little development in this area of mathematics at these ages). An exampleof such a game is Counter Toss, where children are given a counter with different colours on each side (say, red and blue). The child is asked if they can toss a red any time they like (and the very young say that they can) and then to show how they can. After a number of failed tosses, children are asked to say why they cannot always toss a red when they want to. Responses tend to be silence, or a simple 'I don't know'. There is litle development of the concept of chance apparent from responses to this game.

Conclusions

The development to date has provided data on some twelve tasks (the games) and has confirmed that some are very engaging, some are very easy for children as young as three, and others are perplexing for many children.

However, it has been encouraging to note that nearly all children can make a start at the simplest level, and that some children can surmount challenges that many would consider too difficult for them.

We are interested to hear from like-minded researchers and practitioners who wish to contribute ideas, and, or undertake to pilot games with young children.

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