#### **Designing educational mini-games**

Frans van Galen, Vincent Jonker and Monica Wijers Freudenthal Institute for Science and Mathematics Education University of Utrecht, the Netherlands

Children like playing online mini-games, even if these are about a school topic like mathematics, but do they learn from these games? This paper is about mini-games that aim at problem solving. We shall argue that there is a difference between designing software that will be part of the curriculum, and mini-games that will be played by children unsupervised. One of the difficulties is that the computer stimulates children to experiment, but experimenting may also keep children from thinking through a problem.

High expectations exist with respect to the potential use of games for learning (Gee 2003; Kirriemuir 2003; Shaffer 2006). Most of the studies into the effects of games on learning focus on 'big' games, like multiplayer online games (Copier 2007; Squire & Barab 2004); less is known about the effects of online casual games or mini-games (Juul 2007; Prensky 2005). There is not much doubt that mini-games are attractive for children. Children aged 8-12 often play online games on portals like addictinggames.com and funnygames.com. (ISFE 2008; Rohrl 2009; McFarlane 2002) Our website - www.rekenweb.nl or www.thinklets.nl (in English) - with mathematical mini-games attracts thousands of visitors each day, and we know that many of them play the games at home, after school hours (Jonker & Wijers 2008). An important reason for creating this website was to show that mathematics can be fun and we seem to have been successful in that respect. It is also important, however, to know, what children learn by playing these games.

When talking about learning, we want to make a distinction between the acquisition of skills, and the acquisition of mathematical concepts. Most of the existing mathematical mini-games, including several at our website, aim at improving skills, like, for example, proficiency in calculations or in using the multiplication tables. We do not doubt that such skills can be practiced efficiently on the computer and in games (Wastiau, Kearney and Van den Berghe, 2009). The acquisition of mathematical concepts, however, is based on problem solving, at least in our view on mathematics learning. In our approach, based on the theory of Realistic Mathematics Education (Treffers 1987; Gravemeijer 1994; Drijvers & Gravemeijer 2004), the teacher poses problems to the students which challenge them to use previously acquired knowledge and skills in new ways. Discussions about their own solutions and about more general solution methods are believed to be the most important factor in the growth of children's mathematical understanding.

This paper is about games that ask for problem solving. The problem in a mini game can be similar to a problem that is introduced by the teacher and discussed in a whole class discussion. An important difference, however, is that mini-games are mostly played by children on their own, without the supervision of a teacher. So if children learn at all, they do this without the help a teacher can provide, like guiding their thinking in a direction that is mathematically relevant, helping them when they are stuck, or helping them to translate local discoveries into more general mathematical knowledge.

In this paper we focus on two mini-games, called 'Cake' and 'Star' ('Taart' and 'Ster', in Dutch)<sup>1</sup>. They were designed as games to stimulate children to study divisibility and factors. We present data about the use of the games in a situation where they were part of the curriculum, with a teacher present to guide the learning processes of the children. We will contrast this with a situation in which children played a version of the game Cake on their own, observed by an experimenter who did not interfere with their solution process.

Our main conclusion is that unsupervised play asks for a careful design of the mini-game. There is a real danger that children avoid mathematical thinking and stick to using primitive - trial and error - solution methods. We argue that this is related to chracteristics of the computer: a computer offers children the opportunity to experiment, but this may also keep them from reasoning on a higher mathematical level.

#### The mini games Cake and Star

The original version of the mini game Cake is shown in figure 1. To decorate the cake the player chooses an object – e.g. a strawberry - and drags it onto the cake. A click on the decorate button makes the computer put a second strawberry on the cake at a distance specified by 'a number of minutes'. Every next click gives a new strawberry at the same distance from the previous one. The computer rotates the objects, in such a way that if the first strawberry is pointing towards the center of the cake, the next strawberries are also pointing towards the center. A player can choose between various types of fruit, various types of chocolate decoration, and between dots and curves of whipped cream in different sizes. The long lines in the given example are made with separate curves of cream.

The game offers opportunities to all sorts of mathematical discoveries. For example:

1. *Factors of 60.* The distance between the objects on the cake can vary and is expressed in 'minutes'. Similar to a clock, one complete round on the cake is 60 minutes. If a player enters '10 minutes', the computer puts a total of 6 strawberries in a circle on the cake. If the player enters '5 minutes', the computer makes a circle of 12 strawberries. If the number of minutes is a factor of 60, the computer makes exactly one round and then it stops.



Figure 1. The original version of Cake.

2. *Modulo 60.* If the number of minutes is not a factor of 60, the computer puts new objects on the cake in a second or third round and keeps doing so until it ends on a position that is already taken. If, for example, a player chooses 8 minutes, the computer stops after two rounds. This can be explained by the total number of minutes in two rounds (120), which has 8 as a factor.

3. *Geometry*. By choosing a different location for the first object, the orientation of the objects can be varied. In this way it is possible to make complicated line patterns of cream or chocolate out of short curves.

Star, another mini game, is very similar to Cake. In Star dots are placed on a circle – starting at the top - and consecutive dots are connected with a line. The number of marks on the circle and the number of marks to be skipped in every step can be set by the player.



Figure 2. Star.

In the example shown in figure 2 a distance of 25 marks was chosen on a circle with a total of 60 marks, which resulted in many rounds of new dots until the computer stopped at the top again. In certain ways Star is more explicit than Cake, since the connecting lines show the order in which the dots are placed on the circle and the color of the dots changes each round.

# A school experiment with Cake and Star

Using the mini-games Cake and Star in the versions described above, experiments were carried out in grade 5 on two schools in New York<sup>2</sup>. In this paper these experiments will not be discussed in detail, but the work of the students in one of the schools will be used to illustrate that Cake and Star are indeed challenging games for mathematical inquiry.

Starting point in the experiments was that the students would have the freedom to formulate their own research questions. It was one of the reasons for doing these experiments in New York, as most Dutch teachers teach in a more structured way, in which mathematics textbooks play a dominant role.

Both experiments were done in the mathematics lessons of five consecutive days, during which the children worked in pairs or groups of three for most of the time. From time to time class discussions were held about the questions the students had formulated and about what they had discovered so far. At the end of the week a 'math congress' was held with discussions about the posters the children had made.

To illustrate the diversity in the work of the students, we shall discuss the posters of one of these classes.

Factors



Figure 3. A poster about the role of the factors of 60.

There were three groups of students who made a poster about the role of factors of 60. One of these is shown in figure 3. The children in this group searched for numbers of minutes that would result in exactly one round of objects on the Cake. On their poster they have listed the numbers in pairs: a distance of 6 minutes gives 10 pieces (objects) on the cake, whereas a distance of 10 minutes gives 6 pieces. They have discovered that the numbers that generate one full round are factors of 60 and they note that they have tested that 'minutes x pieces= 60'.

#### Clockwise and counter-clockwise



*Figure 4.* A poster by children who studied the direction in which pieces are placed on the cake.

Another group has studied the direction in which the objects are placed on the cake, clockwise or counter-clockwise (see figure 4). They had noticed that both with 15 and with 45 minutes the computer puts four pieces on the cake, but in a different order. The same applies to number pairs like 11 and 49, that each puts 60 pieces on the cake.

In the math congress a child of this group used the Star program to explain the pattern they had discovered. This led to a discussion that ended with the conclusion that the computer only appears to go counter-clockwise with numbers like 45, but actually makes big clockwise jumps. The class also seemed to understand why the numbers in these pairs add up to 60.

#### Prime numbers

Two students did study the role of prime numbers in the mini games. Their poster shows a list of all prime numbers up to 60 with the number of rounds before the computer stops putting new fruit on the cake. According to them choosing 7 minutes gives a total number of 6 rounds, whereas 59 minutes gives 58 rounds. They write:

'We noticed that prime numbers and numbers not factors of 60 will take one less than the number itself in rounds to get around. To 60. Which is a whole. At first we thought it was just a coincidence, but after a while we saw it was true.'

In the math congress it turned out that these students did not count the last round, as they only counted the number of times that the computer passed the starting point when putting new objects on the cake. The discussion about what counted as one round took a lot of time in the math congress and the question how their pattern could be explained was not discussed.

### Fractions

John and Ashley traction ounder 10 6 6 4 15 4 20 3 50 omin. is numerator on a clock is the round the denominator so that means the round and the pieces is 15

*Figure 5. A poster that describes how the relation between number of rounds and the number of pieces can be described by a fraction.* 

Two other students studied the relation with fractions. They had noted that choosing 10 minutes gives 6 pieces and those pieces divide the cake in parts of 1/6. Their explanation was that 10 minutes is 1/6 of 60. They had also studied numbers that lead to more than one round. As an example they wrote on their poster that 8 minutes is 8/60 on the cake, which can be simplified to 2/15, meaning that the computer will need 2 rounds (numerator) and will give 15 pieces (denominator) in total. They could not explain fully why this rule holds.

### Conclusions from the school experiment

Although the students could not fully explain all the patterns they had found, we may conclude that the students did meaningful mathematical inquiries; they discovered patterns, tried to explain these patterns and succeeded in doing so in most cases.

A point of discussion after the experiments was whether the results gave reasons to change the mini-games. Especially the fact, that in Cake the computer could make more than one round had complicated the study of factors and divisibility for the children. After the first two days, therefore, the teacher had decided to steer all students towards an inquiry into the question what numbers would lead to just one round. Adjustments in the design of Cake were believed to help students to focus on the role of factors.

# Copy the Cake

The school experiment shows the power of Cake and Star as mini-games for mathematical inquiry. A school situation, however, is quite different from a situation in which children play these mini games unsupervised. For an unsupervised situation the question how open the task should be, will have a different answer, since in that situation all guidance has to come from the game itself.

In the remainder of this article we shall focus on Cake, the most popular game of the two. Cake was published in 2001 and has always been in the top ten of most popular games on www.rekenweb.nl . The game attracts about 600 visitors per day from the Netherlands and Flanders, which is a respectable number. Ever since the game has been on the internet, however, we have received signals that students pay little attention to the mathematical aspects of the task. One could argue that this was predictable, as the game does not explicitly ask children to study these aspects. In this respect the situation of playing the game unsupervised is very different from the situation in the school experiments, where the teacher asked the students to look for patterns and formulate research questions. The game offers children a rich situation to explore, but it does not specify the direction of the exploration.

For these reasons we decided to create an adjusted version of Cake in which children are asked to copy a cake. This is still very different from asking children to study patterns, but it forces children to think about the mathematics in the task, at least up to a certain level. We also made adjustments that would help children to focus more on divisibility and factors.

Figure 6 shows the new mini-game Copy the Cake (In Dutch: 'Taart Namaken'). The game does not check whether a cake has been copied correctly, as this would involve complicated programming; we leave this to the judgment of the children.

In the new version the way objects are put onto the cake is also different. In the original version the player had to drag one object to the cake, set the number of minutes and then click the decorate button for as many times as the number of objects he or she wanted on the cake. In the new version the game puts new objects automatically on the cake, after the first object is dragged onto it. If, for example, the distance has been set to 6 and a strawberry has been dragged onto the cake, the game automatically puts a 10 strawberries in a circle on the cake.



*Figure 6. The new version of Cake, in which children have to copy the cake that is shown at the left.* 

An even more important change is that the game stops placing new objects on the cake once it has made a full round and is back at 60. What we have called the modulo aspect before, does not apply anymore. In Copy the Cake a distance of 15 gives four objects on the cake (at 0, 15, 30 and 60 minutes). A distance of 45, however, puts only one object on the cake. The assumption is that it is better if children concentrate on divisibility and factors of 60 before they experiment with modulo situations. For the latter the Star program could be used.

Minor changes are that 60 red marks are shown around the cake and the terminology of minutes has been changed to: 'put them after .. marks'. The layout has been adjusted, giving the box to enter the number that specifies the distance, a more conspicuous place. Also with each new object on the cake a number appears at the bottom of the screen. A distance of 15 returns the string '0 15 30 45'.

### **Observations with Copy the Cake**

With this new version of Cake a series of eight problems was constructed and this series of problems was presented to 13 students of 11 and 12 years old. Each student was observed while doing the problems individually. The research question in this experiment was to find out if the adjustments would help students discover how division can be used to decide beforehand how many objects will appear on a cake. The eight problems were presented in the following order:

- The first assignment was not to copy a cake, but to make a cake that the student would call interesting. This assignment was intended to establish if children already knew the Cake game and if they had experimented before with changing the number of minutes or marks (the distance).

- In the second problem a cake had to be copied with all objects at a distance of 5 marks. This is the default value when the game starts.
- In the third problem a text below the cake to be copied told the student that the number in the box had to be changed. In this case the correct number was 4, not 5.
- In the next three problems other numbers had to be used, like 6, 10, 12, 15 and 30 in order to copy the cake correctly. In the fourth and fifth problem a text told the student that there were 60 red marks around the cake. The fifth problem is shown in figure 6.
- In problem 7 the student had to copy an 'untidy' cake with strawberries spread unevenly over the cake. The correct distance here was 18 marks.
- In problem 8, finally, the model remained empty, but the text asked for a cake with 6 strawberries, 5 pieces of orange, 4 pieces of kiwi fruit, 3 pieces of pineapple and 2 cherries.

After the series of eight problems was completed, the experimenter interviewed the children about the relation between the distance between objects, the number of objects and the total number of marks around the cake.

### Research method

13 children were observed when they tried to solve these problems. They were 11 and 12 years old. All children said that they knew Cake and had played the game before, but for most of them this had been some time ago, as the game had been more popular in lower grades.

We used a laptop and a computer program - iShowU for Apple computers that registered every mouse click of the student. The program also recorded audio and it used the webcam to videotape the face of the student. The children were tested individually. This was done at school but outside of the classroom. The observations lasted 12 to 27 minutes. The researcher was sitting beside the child, but did not interfere. When the behavior of the child made it plausible that the child had discovered how a certain distance would lead to a specific number of objects, the researcher asked the child to explain his or her choice of that number for the distance. In many cases the answer proofed that the child had indeed discovered the division rule, but it also happened that a child said that it had chosen 10, for example, because the number 10 had given 6 objects before.

### Results

# The first assignment: 'Make an interesting cake'

The first assignment was intended to check whether children were familiar with the Cake and had experimented with changing the distance between objects in the earlier version of the game. Of the 13 subjects 11 made a cake with all distances set to the default value of 5; only two of them spontaneously changed this number. Some children remarked that they had had been puzzled by the original cake; they had never understood what making cakes had to do with mathematics.

## Discovering the relation with division

Reviewing the recorded sessions made it possible to pinpoint the moment where students made the step from a strategy of trial and error to a strategy based on division. The following fragment shows how one of the students comes to see 'how it works'.

Problem 3. Lisa reads in the text that she has to change the number in the box. She tries 8 and says: 'I believe I understand; if you change the number, it comes so many times.' She tries 16 - obviously not correct - and then 4, which satisfies her. When the experimenter asks why she had chosen 4, she says: 'I do not know exactly how it works, but I gambled.'

Problem 4. She reads the text: 'There are 60 red marks around the cake. Do you see them?' She says 'Yes' and continues: 'Maybe, you have to do something with division. Wait, maybe I know how it works.' And after she has seen that 10 is the correct number: 'I think I understand.'

When she is doing the next problem, she gives an explicit explanation: '60 divided by 4 makes 15.'

Table 1 summarizes at which moment the children in the study made this discovery. Most striking in these results is that almost half of the students ( 6 out of 13) did *not* discover the division strategy while working on the problems. These 6 children solved the problems with a strategy of trial and error: changing the number in the box, making it larger or smaller, until the result satisfied them. Quite often also, they remembered from an earlier problem that, say, 10 would lead to 6 objects on the cake. In the interview afterwards, however, it was sometimes enough to refer to the number of marks on the cake to let the children conclude that division could have been used.

Number of students	Point at which the relation with division was discovered
2	problem 4
2	problem 5
2	problem 6
1	problem 8
6	interview afterwards

*Table 1. Point at which students discovered the possibility to use division.* 

An important stumbling-block appeared to be that students did not read the texts below the model cakes. The text for problem 4 said: 'There are 60 red

marks around the cake. Do you see them?' and the text for problem 5 was: 'Remember the 60 marks on the cake'. Even more alarming, however, is that the students did not seem to take in the meaning of the sentence around the number box, which was: 'put them every .... marks' ('leg om de ... streepjes'). When they explained what they were doing they did not refer to the marks. Also the fact that students hardly ever counted the number of marks between two consecutive objects points to this. It is as if the students only used the box to change the distance, without trying to interpret the number in any other way than: it makes the distance between objects larger or smaller.

All in all, the results are meager: 6 out of 13 students do not discover the role of division without help. They solve most problems correctly, but use a primitive, trial and improve strategy to set the correct distance:

- They know that changing the number in the box changes the distance between objects. Most of the students know that a larger number gives a larger distance, but some students find it difficult to remember even that.
- They remember that specific numbers lead to a certain number of objects on the cake, but just as isolated facts.

### Conclusions from the observation experiment

The fact that so few of the students discover the possibility of using division to calculate the number of objects or the distance they need to set, raises some serious questions. Of course it can be argued that the series of problems in the experiment was not helpful enough and undoubtedly some improvements - like showing the text about the 60 marks in a larger font and in red, for example - will lead to better results, but two points of concern remain.

The first one is that students do not seem to feel the need to go beyond the primitive strategy they have discovered. As long as this strategy leads to results, they do not question their approach. Furthermore they do not try to interpret the number in the box in a way that goes beyond: 'a larger number gives a larger distance between objects'.

The second point of concern is that our task was only aiming at very basic mathematical relations: 60 divided by the number of marks between objects gives you the total number of objects; or the other way around: 60 divided by the number of objects you want, gives you the distance that should be set. Much more interesting relations are:

- What numbers will lead to 'tidy' cakes and what numbers will lead to 'untidy' cakes?
- How can one find all numbers that result in 'tidy' cakes?
- Why is it possible to conclude from the fact 'every 10 marks' gives 6 objects, that 'every 6 marks' will give 10 objects?

Only if students try to answers such questions, one can say that they study 'divisibility'. If it is already this difficult to let children see the relation

between the total number of marks and the number of marks between objects, one may wonder how it will be possible to stimulate children to think about questions that go beyond this.

#### Discussion

Our goal in this paper was to show the difference between designing minigames that will be used in a class setting supervised by a teacher, and designing educational mini-games for use outside classroom settings. Designing an educational mini-game is more demanding if the mini-game will be played by children unsupervised, as this means that all guidance has to be built into the game. To complicate the task even further, the data from our observations show that experimenting may withhold the children from thinking through a problem more fundamentally.

With respect to the last point, our findings did not really surprise us, as we had before similar experiences with other mini-games. One of these is Island (van Galen, 2000; Figueiredo, Gravemeijer & van Galen, 2007)<sup>3</sup>. In this game children have to find a treasure by finding out, among other things, from which point at sea a picture was taken of the three towers on the island. When we observed children playing this game, we saw that most of them just tried out many spots until they were satisfied with the result. The choices they made did not seem to be well thought over. Significant in this respect is that their discussions - the students worked in pairs - remained restricted to remarks like: 'Try this spot' and 'Click here'. As the computer gives feedback on every click - by showing what a picture from that position would look like - there seemed to be no need for more profound deliberations. This stood in sharp contrast to another situation in the experiment, where we gave students similar problems, but now on paper and with no feedback at all. This situation stimulated children to confer with another extensively, and to give all sorts of arguments for the choices they made.

These finding suggest that compared with more traditional ways of presenting mathematical problems, the computer seems to have both advantages and disadvantages. The advantage is that children can freely experiment; the disadvantage is that experimenting seems to withhold children from thinking through the problems in a more fundamental way.

With respect to the difference between use in school and unsupervised play, we see two possible approaches to structure the mini-game. The first one is to build questions and hints into the mini-game, probably the same questions and hints a teacher might provide. A very powerful question could be to ask at a certain point: 'Can you tell how many red marks there are around the cake?' Another question could be: 'Make a list of all numbers that will give you a tidy cake' Adding such questions, however, will turn the mini game into a task that resembles the tasks that children have to do at school, and will make Cake less attractive as a game.

Another approach could be to discourage experimentation, forcing children to think through an answer before they try it. This can be done by adding game-

like options like score-points, levels etc. Players could be rewarded, for example, with extra points if they do not use the undo button, or they may be allowed only three trials. This approach would be more in line with what children expect in a mini-game, and it is also what children know from playing larger games. It remains to be seen, however, if such measures will be effective enough.

Whatever approach is chosen, it will result in a task that is much less open than the original version. One might argue with good reason that the original version was too open; in fact it did not even state that there was a problem to be solved. We would like to avoid, however, that the mini-game takes the children by the hand and forces them to make very small steps, one at a time.

The theory of Realistic Mathematics Education favors the use of carefully designed, but open mathematical problems. The problems should allow for students to solve them in different ways, as it is believed that comparing solutions and reflecting on the mathematics behind solutions will foster students' growth in mathematical thinking. It is not yet clear what place online mini-games might conquer within mathematics education. Our observations show that a rich mathematical context does not translate automatically into a mini-game that stimulates mathematical thinking. This suggests that we need to work on creating a specific set of design principles for mini-games.

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<sup>&</sup>lt;sup>1</sup> The original versions of the games were designed by Frans Moerlands and Peter Boon. The Dutch version can be found at <u>www.rekenweb.nl</u>, the English version can be found at <u>www.thinklets.nl</u>. Cake has also been called 'Pie'.

<sup>&</sup>lt;sup>2</sup> The schools participated in 'Math in the City' of City College New York. The experiments were carried out in collaboration with Catherine Fosnot.

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